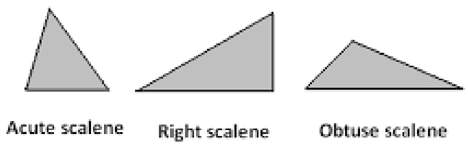
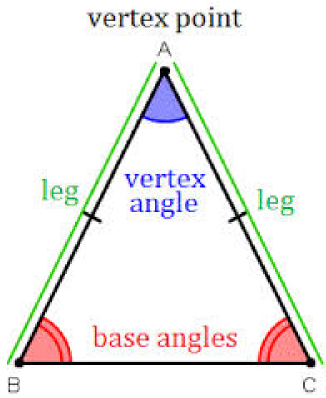
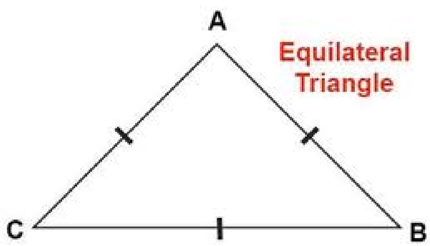
Lesson 3: Angle Sum of a Triangle (Day 1)

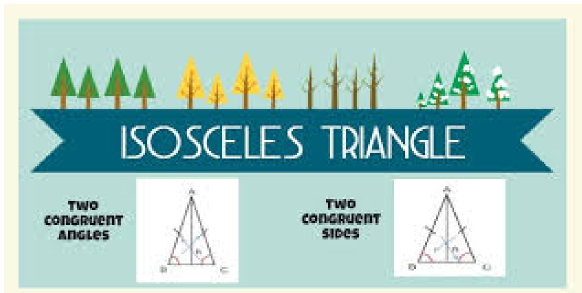
- The Angle Sum Theorem for triangles states that the sum of the interior angles of a triangle is always 180° ($\angle$ sum of $\triangle$).
- It does not matter what kind of triangle (i.e., acute, obtuse, right) when you add the measure of the three angles, you always get a sum of 180.



Scalene Triangles Types







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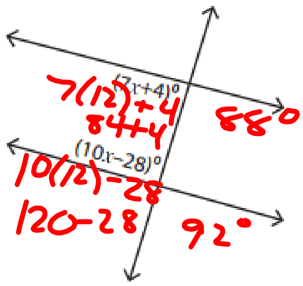


Geometry Fact: Supplementary ∡'s

Equation:

$(7x+4)+(10x-28)=180$

$7x+4+10x-28=180$

$17x-24=180$

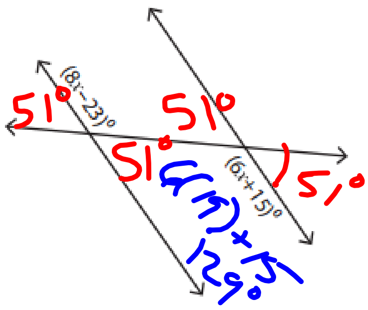
$+24 \quad +24$

$17x=204$

$\frac{17x}{17}=\frac{204}{17}$

$x=12$

3.



Geometry Fact: Alt. Interior ∡'s

Equation:

$8x-23=6x+15$

$-6x \quad -6x$

$2x-23=15$

$+23 \quad +23$

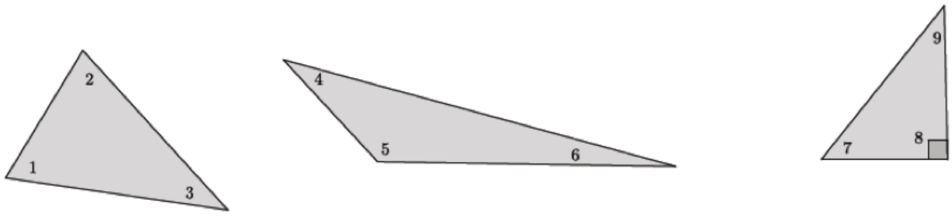
$2x=38$

$\frac{2x}{2}=\frac{38}{2}$

$x=19$

Classwork

Concept Development



$$\angle 1 + \angle 2 + \angle 3 = \angle 4 + \angle 5 + \angle 6 = \angle 7 + \angle 8 + \angle 9 = 180$$

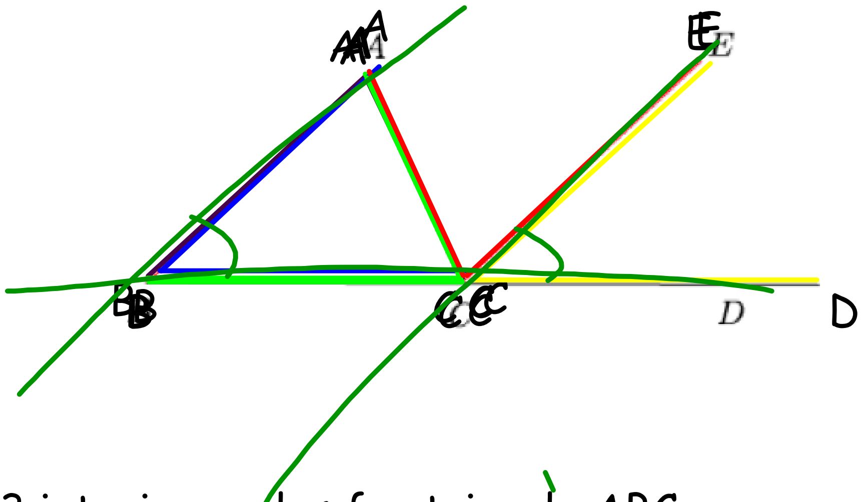
Note that the sum of angles 7 and 9 must equal 90˚ because of the known right angle in the right triangle.

We want to prove that the angle sum of any triangle is 180˚. To do so, we will use some facts that we already know about geometry:

- A straight angle is 180˚ in measure.
- Corresponding angles of parallel lines are equal in measure (corr. $\angle s$, $\overline{AB} \parallel \overline{CD}$).
- Alternate interior angles of parallel lines are equal in measure (alt. $\angle s$, $\overline{AB} \parallel \overline{CD}$).

Exploratory Challenge 1

Let triangle ABC be given. On the ray from B to C, take a point D so that C is between B and D. Through point C, draw a line parallel to AB as shown. Extend the parallel lines AB and CE. Line AC is the transversal that intersects the parallel lines.



The 3 interior angles for triangle ABC are....

The 3 angles that make up line BCD are....

Of these 6 angles, which angles appear to be the same measure?

a. Name the three interior angles of triangle ABC.

b. Name the straight angle.

c. What kinds of angles are $\angle ABC$ and $\angle ECD$? What does that mean about their measures?

d. What kinds of angles are $\angle BAC$ and $\angle ECA$? What does that mean about their measures?

e. We know that $\angle BCD = \angle BCA + \angle ECA + \angle ECD = 180°$. Use substitution to show that the three interior angles of the triangle have a sum of 180°.

a. **Name the three interior angles of triangle ABC.**

$\angle ABC, \angle BAC, \angle BCA$

b. **Name the straight angle.**

$\angle BCD$

Our goal is to show that the three interior angles of triangle ABC are equal to the angles that make up the straight angle. We already know that a straight angle is 180° in measure. If we can show that the interior angles of the triangle are the same as the angles of the straight angle, then we will have proven that the interior angles of the triangle have a sum of 180°.

c. **What kinds of angles are $\angle ABC$ and $\angle ECD$? What does that mean about their measures?**

$\angle ABC$ and $\angle ECD$ are corresponding angles. Corresponding angles of parallel lines are equal in measure. (corr. $\angle$s, $\overline{AB} \parallel \overline{CE}$)

d. **What kinds of angles are $\angle BAC$ and $\angle ECA$? What does that mean about their measures?**

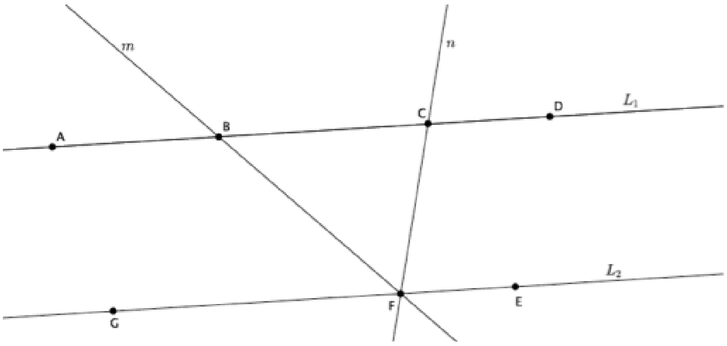
$\angle BAC$ and $\angle ECA$ are alternate interior angles. Alternate interior angles of parallel lines are equal in measure. (alt. $\angle$s, $\overline{AB} \parallel \overline{CE}$)

e. **We know that $\angle BCD = \angle BCA + \angle ECA + \angle ECD = 180^\circ$. Use substitution to show that the three interior angles of the triangle have a sum of 180°.**

$\angle BCD = \angle BCA + \angle BAC + \angle ABC = 180^\circ$ ($\angle$ sum of $\triangle$).

Exploratory Challenge 2

The figure below shows parallel lines L_1 and L_2. Let m and n be transversals that intersect L_1 at points B and C, respectively, and L_2 at point F, as shown. Let A be a point on L_1 to the left of B, D be a point on L_1 to the right of C, G be a point on L_2 to the left of F, and E be a point on L_2 to the right of F.



a. Name the triangle in the figure.

△

b. Name a straight angle that will be useful in proving that the sum of the interior angles of the triangle is 180°.

c. Write your proof below.

a. **Name the triangle in the figure.**

ΔBCF

b. **Name a straight angle that will be useful in proving that the sum of the interior angles of the triangle is 180°.**

$\angle GFE$

As before, our goal is to show that the interior angles of the triangle are equal to the straight angle. Use what you learned from Exploratory Challenge 1 to show that interior angles of a triangle have a sum of 180°.

c. **Write your proof below.**

The straight angle, $\angle GFE$ is comprised of angles $\angle GFB$, $\angle BFC$, $\angle EFC$. Alternate interior angles of parallel lines are equal in measure, (alt. $\angle s$, $\overline{AD} \parallel \overline{CE}$). For that reason, $\angle BCF = \angle EFC$ and $\angle CBF = \angle GFB$. Since $\angle GFE$ is a straight angle, it is equal to 180°. Then $\angle GFE = \angle GFB + \angle BFC + \angle EFC = 180^\circ$. By substitution, $\angle GFE = \angle CBF + \angle BFC + \angle BCF = 180^\circ$. Therefore, the sum of the interior angles of a triangle is 180. ($\angle$ sum of $\triangle$).

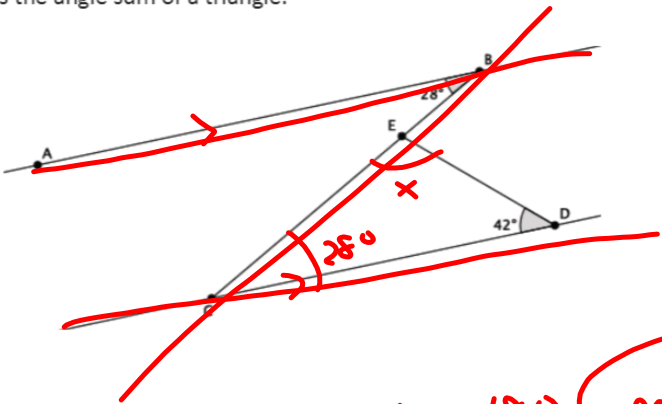
Lesson Summary

All triangles have a sum of interior angles equal to 180˚.

The proof that a triangle has a sum of interior angles equal to 180˚ is dependent upon the knowledge of straight angles and angles relationships of parallel lines cut by a transversal.

Problem Set

1. In the diagram below, line AB is parallel to line CD, i.e., $L_{AB} \parallel L_{CD}$. The measure of angle $\angle ABC = 28°$, and the measure of angle $\angle EDC = 42°$. Find the measure of angle $\angle CED$. Explain why you are correct by presenting an informal argument that uses the angle sum of a triangle.



$28 + 42 + x = 180$

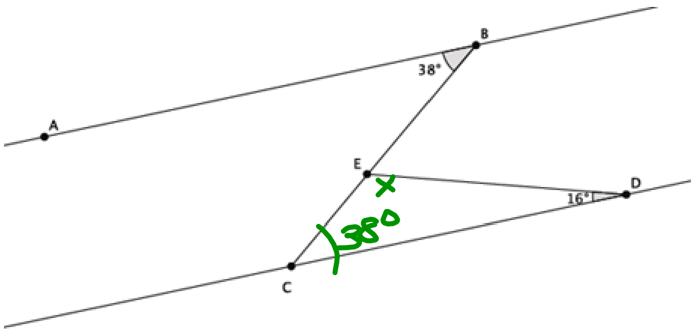
$70 + x = 180$

$-70 \quad -70$

$x = 110$

$m\angle CED = 110°$

2. In the diagram below, line AB is parallel to line CD, i.e., $L_{AB} \parallel L_{CD}$. The measure of angle $\angle ABE = 38°$ and the measure of angle $\angle EDC = 16°$. Find the measure of angle $\angle BED$. Explain why you are correct by presenting an informal argument that uses the angle sum of a triangle. (Hint: find the measure of angle $\angle CED$ first, then use that measure to find the measure of angle $\angle BED$.)



$m\angle BED = 126°$

$x + 38 + 16 = 180$

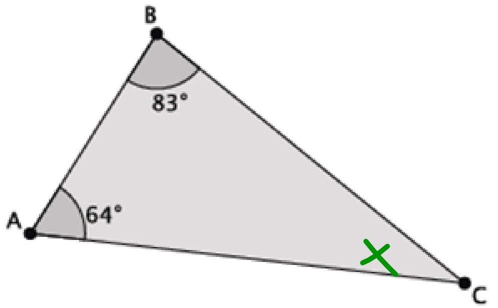
$x + 54 = 180$

$-54 \quad -54$

$x = 126°$

Supplemental Practice

1. What is the measure of $\angle ACB$?



$83 + 64 + x = 180$

$147 + x = 180$

$-147 \quad\quad -147$

$x = 33°$